Group Theory in the Bedroom, and Other Mathematical Diversions

by Brian Hayes

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REVIEWED BY JEANINE DAEMS AND IONICA SMEETS[1]

When we first heard about *Group Theory in the Bedroom*, we became very enthusiastic. We were reminded of *Mathematics and Sex* by Clio Cresswell. We enjoyed that book a few years ago, but it contained too little mathematics to our taste (and surprisingly little about sex, for that matter). The title of Hayes's book sounded very promising: More serious mathematics in the bedroom! However, this bedroom does not appear until the final chapter of the book, which deals with mattress flipping. And in that chapter, the author soon turns to group theory in the garage. We understand why the author chose this eye-catching title, even though it does not really cover the contents.

In the early 1980s, Brian Hayes was working as an editor at *Scientific American* magazine. The magazine wanted to launch a new monthly column called Computer Recreations, and although he had no real knowledge of computers at the time, Hayes volunteered to write it. But he managed, and as he writes in the preface, "I discovered that the computer is not like the violin: It doesn't take inborn genius or a lifetime of practice to get sweet music out of it." Hayes wrote the column for just a few months, but his newly developed interest stayed, and later he wrote some pieces for *Computer Language* and *The Sciences*. Since 1993, he has written another column for *American Scientist*. The essays in *Group Theory in the Bedroom* are all reprints, with one appearing in *The Sciences* and the rest appearing in *American Scientist*. Some of the essays are over 10 years old and show their age. Hayes has added a section called *Afterthoughts* to each essay, in which he describes more recent developments and discusses some of the reactions he got from readers after first publication.

The topics of the essays are quite diverse, although all of them have an algorithmic and/or mathematical flavor. Subjects include: The astronomical clock of Strasbourg Cathedral; randomness; statistics of wars; the history of gears and their relation to computing; the ternary system; and, obviously, group theory. Hayes is no more a mathematician than a computer scientist, but his fascination for the subject shows and he knows his audience well. "I'm not a mathematician, but I've been hanging around with some of them long enough to know how the game is played. Once you've solved a problem, the next step is to generalize it beyond all recognition."

Your two reviewers could not agree on one favorite chapter. Jeanine really enjoyed The Clock of Ages. This essay was written at the end of 1999, when people were preparing themselves for the upcoming New Year. "As the world spirals on toward 01-01-00, survivalists are hoarding cash, canned goods, and shotgun shells. It's not the Rapture or the Revolution they await, but a technological apocalypse. Y2K!" Of course, the apocalypse did not happen on January 1, 2000. But how could the computer programmers of the 1960s and 1970s fail to look beyond 1999? Hayes gives them the benefit of the doubt. No programmer back then would think his programs would still be in use by the year 1999. The important questions of this chapter are whether there is any sense in building things to last, and to what extent?

A very remarkable example of something that was definitely built to last for a very long time is the astronomical clock of Strasbourg Cathedral (see "On Picturing the Past: Arithmetic and Geometry as Wings of the Mind" by Volker Remmert, this magazine, Vol. 31, No. 3). In its present form, it was started up in 1574. It is more an astronomical and calendar computer than a clock. It keeps track of a host of objects and events: The positions of 5,000 stars, the six inner planets, the current phase of the moon, sidereal time, local solar time, local lunar time, mean solar time, the present year, the day of the year (including February 29th in leap years), and the "movable feasts" of the ecclesiastical calendar among others. All this is done with gears, with the clock's error being less than a second per century. Schwilgué, the maker of the clock, was thinking long-term. He included parts in the leap-year mechanism that engage only every 400 years and that were first tested in the year 2000. The clock can represent the years until 9999, but Schwilgué suggested that after that there just should be a "1" painted on the left of the thousands digit. The existence of a clock like this raises the question: Is the building of multimillennial machines a good idea? Hayes is doubtful.

Ionica found the story about the clocks rather dull, but she loved Inventing the Genetic Code. Hayes describes how physicists and mathematicians in the early 1950s tried to break the code of DNA right after biologists discovered that the language of the double helix consisted of just four letters: A, T, G and C. The big question was how these four letters coded 20 different amino acids. Many beautiful coding schemes were invented, very efficient codes with nice symmetries and error-correcting possibilities. With the discovery of the actual encoding scheme, Hayes admits that he was rather disappointed by nature's real solution. He jokes that we might have been better off with the genetic code devised by one of the mathematicians: "Life would be a lot more reliable if Solomon Golomb were in charge."

We both liked the final chapter of the book, the one that gave the book its title. It presents a nice way to

[1]Jeanine Daems and Ionica Smeets–often referred to as The Math Girls (*wiskundemeisjes* in Dutch)–are two female Dutch Ph.D students of mathematics who started, in March 2006, a hugely successful website devoted to cultural aspects of mathematics: http://www.wiskundemeisjes.nl. They have also appeared as celebrities on TV shows and in other media in The Netherlands.–**O. P.**

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explain group theory. Group theory helps when the author can't sleep at night: "Having run out of sheep the other night, I found myself counting the ways to flip a mattress." In the long intervals between seasonal flips, he always forgets which way he flipped the mattress the last time, so he asks if there is a so-called golden rule for mattress flipping, i.e., a consistent rule telling you what to do that results in the mattress cycling through all its possible configurations. Using the transformations one could perform while flipping a mattress into another proper position and the structure of the mattress's symmetry group, he answers the question.

Then the author turns to group theory in the garage, asking for a similar golden rule for rotating the tires of a car. The structures of the Klein 4-group and the cyclic 4-group are explained in the process. Hayes even discusses ways to flip a hypothetical cubical mattress, daring the readers to do the same with a 4- or even more-dimensional cube. However, in the "Afterthoughts," it becomes clear even this is not enough generalization for mathematicians. Readers have written him about polygon-shaped mattresses, circular mattresses and even a Möbius mattress.

It is not very clear what kind of audience this book aims at; we think a potential reader should be familiar with algorithmic thinking and some mathematics. Hayes is not wholly consistent in the prior knowledge he assumes his readers have. Most explanations of the mathematics and algorithms are very clear, but some may be too brief for nonmathematicians. Luckily, his examples are very well chosen and most of them are appealing. For example, "To appreciate the value of randomness, imagine a world without it. What would replace the referee's coin flip at the start of a football game?"

Our main criticism is that 12 of these essays is too much of the same. Hayes likes computer simulations a bit more than we do, and after a few chapters we started groaning when he proudly presented another homemade diagram. At one point we were also slightly annoyed with his lack of understanding of the mathematics. In the (highly enjoyable) chapter The Easiest Hard Problem, Hayes counts the number of perfect partitions of a set of n integers. He nicely illustrates this problem with picking teams for a ball game where you want the teams to be as equal as possible. In mathematical terms: Given a set of integers, you want to divide it into two subsets that have the same sum of values. If the sum of all values in the original set is odd, then this is impossible. In this case, a perfect partition is given by two subsets whose sums differ by exactly one. Of course, Hayes could not resist doing a bunch of computer experiments, and he was surprised that "sets whose sum is an odd number have about twice as many partitions, on average, as similar sets whose sum is an even number." He emailed some top-range mathematicians about this "strange" phenomenon and they kindly helped him solve this "mystery."

This example is the exception, though. Hayes took a lot of effort to delve into the subjects, and his view of mathematics from the outside is refreshing. In many cases, it is charming to see his personal struggle with the material.

We recommend this book to people who are already know a bit about mathematics and algorithms and who love to read about problems and questions in the real world and its generalizations. We think that, for those readers, the book is extremely suitable for reading in the bedroom. So, at least in this sense, the title fits!

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