

Language: E1	nglish
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Day: 1

Wednesday, July 15, 2009

Problem 1. Let *n* be a positive integer and let a_1, \ldots, a_k $(k \ge 2)$ be distinct integers in the set $\{1, \ldots, n\}$ such that *n* divides $a_i(a_{i+1}-1)$ for $i = 1, \ldots, k-1$. Prove that *n* does not divide $a_k(a_1-1)$.

Problem 2. Let ABC be a triangle with circumcentre O. The points P and Q are interior points of the sides CA and AB, respectively. Let K, L and M be the midpoints of the segments BP, CQ and PQ, respectively, and let Γ be the circle passing through K, L and M. Suppose that the line PQ is tangent to the circle Γ . Prove that OP = OQ.

Problem 3. Suppose that s_1, s_2, s_3, \ldots is a strictly increasing sequence of positive integers such that the subsequences

 $s_{s_1}, s_{s_2}, s_{s_3}, \dots$ and $s_{s_1+1}, s_{s_2+1}, s_{s_3+1}, \dots$

are both arithmetic progressions. Prove that the sequence s_1, s_2, s_3, \ldots is itself an arithmetic progression.



Language:	English
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Day: 2

Thursday, July 16, 2009

Problem 4. Let ABC be a triangle with AB = AC. The angle bisectors of $\angle CAB$ and $\angle ABC$ meet the sides BC and CA at D and E, respectively. Let K be the incentre of triangle ADC. Suppose that $\angle BEK = 45^{\circ}$. Find all possible values of $\angle CAB$.

Problem 5. Determine all functions f from the set of positive integers to the set of positive integers such that, for all positive integers a and b, there exists a non-degenerate triangle with sides of lengths

a, f(b) and f(b + f(a) - 1).

(A triangle is *non-degenerate* if its vertices are not collinear.)

Problem 6. Let a_1, a_2, \ldots, a_n be distinct positive integers and let M be a set of n-1 positive integers not containing $s = a_1 + a_2 + \cdots + a_n$. A grasshopper is to jump along the real axis, starting at the point 0 and making n jumps to the right with lengths a_1, a_2, \ldots, a_n in some order. Prove that the order can be chosen in such a way that the grasshopper never lands on any point in M.